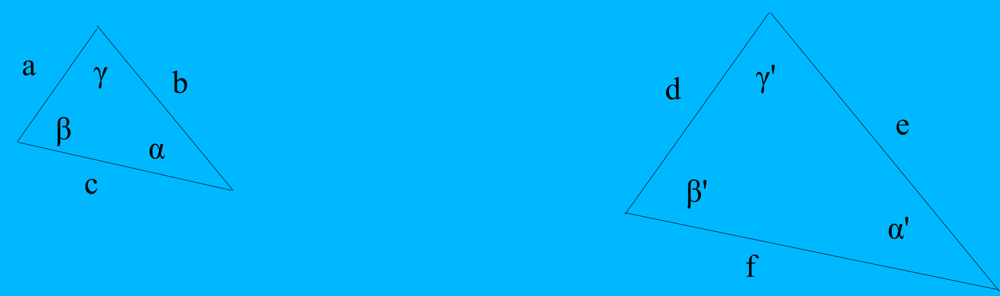
Similar Triangles

Two triangles are similar if:
1) the corresponding angles of the first in the second are the same, or
2) the ratios of corresponding sides of the two triangles are equal



1) $\alpha = \alpha'$ $\quad \beta = \beta'$ $\quad \gamma = \gamma'$

2) $a/d = b/e = c/f$

$d/a = e/b = f/c$

From these equalities we have shorter requirements.

Two triangles are similar if any of the following is true:

1) two angles of one equal the corresponding angles of the other
$\alpha = \alpha'$ $\quad \beta = \beta'$

2) the ratio of two sides of one triangle equals the ratio of the corresponding sides of the other and the included angles are equal
$b/c = e/f$ and $\alpha = \alpha'$

3) the ratio of two sides of one triangle equals the ratio of the corresponding sides of the other and the agles opposite the largest sides are equal
$a/b = d/e$ and $\beta = \beta'$

4) the ratios of two pairs of sides of one are equal to the ratios of two pairs of corresponding sides of the other
$b/c = e/f$ and $a/b = d/e$

Congruent Triangles

Two triangle are congruent if they have the same shape and the same size.

Any one of the following must be true for congruent triangles:

1. Two angles of one triangle equal their corresponding angles of the second and the corresponding included sides are equal. ("asa": angle-side-angle)

2. Two sides of one triangle equal their corresponding sides of the other and the corresponding included angles are equal. ("sas": side-angle-side)

3. Two sides of one triangle equal their corresponding sides of the other and the corresponding angles opposite the larger sides are equal. ("ssa": side-side-angle)

4. Sides of one triangle equal corresponding sides of the other triangle. ("sss": side-side-side)

1. (asa) $\alpha = \iota$ $c = d$ and $\gamma = \theta$

2. (sas) $b = e$ $\gamma = \theta$ and $c = d$

3. (ssa) $a = f$ $c = d$ and $\beta = \kappa$

4. (sss) $a = f$ $b = e$ and $c = d$

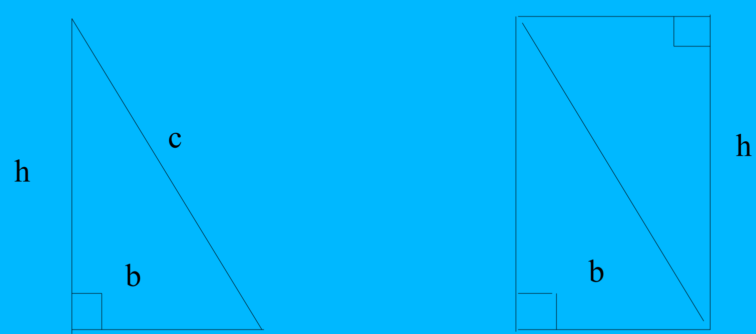
TRIANGLES

Perimeter: distance around the triangle, the perimeter of a triangle is the sum of the lengths of its sides.

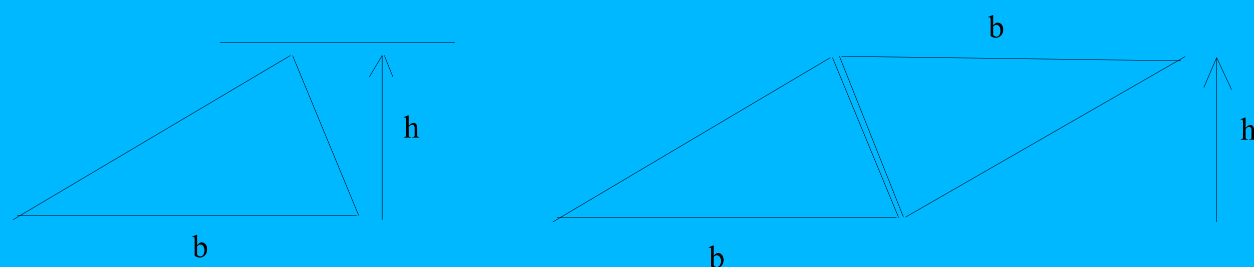
Area: the area A of a triangle is ½ its base b multipled by its height h.

| perimeter = b + h + c |

| A = ½ b · h |



This rectangle has area equal to b· h. Since the triangle is half this rectangle then its area must be ½ b · h.

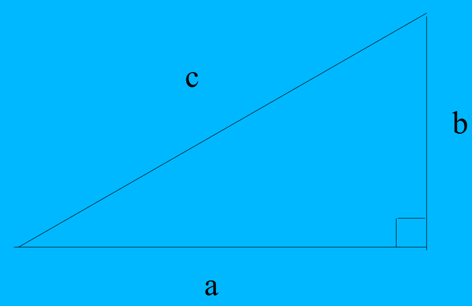


The area of a parallelogram is its base b multiplied by its height h. Every triangle can be converted into a parallelogram, therefore the area of any triangle is ½ the area of its converted parallelogram, that is, ½ b · h.

Theorem of Pythagoras

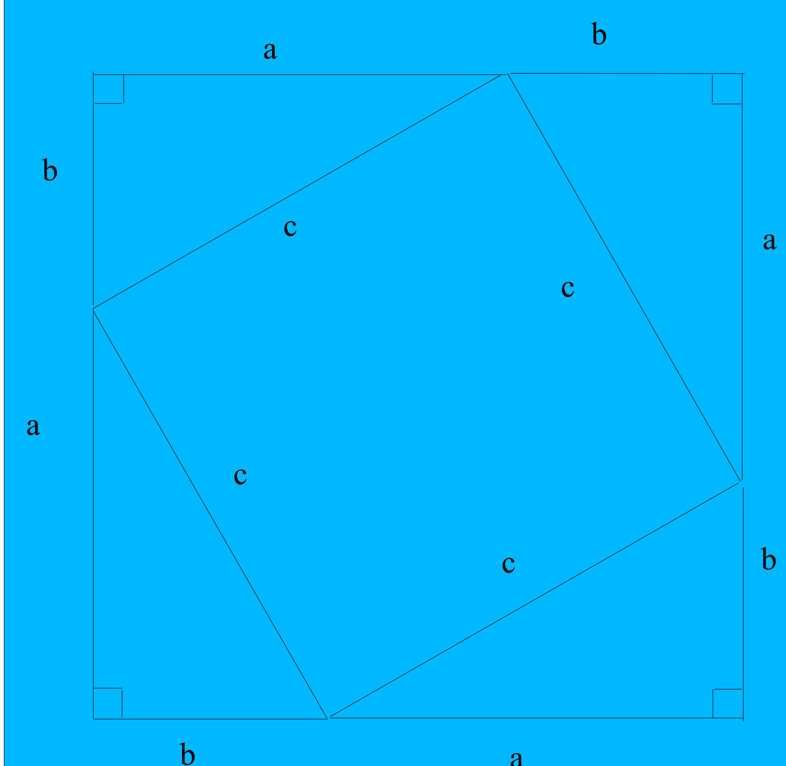
The sum of the squares of the legs of a right triangle equals the square of the hypotenuse.

$$\mathbf{a^2 + b^2 = c^2}$$



a and b are the "legs"

c is the " hypotenuse"



proof:

The area of the outer square equals the combined areas of the 4 right triangles plus the area of the square with sides c.

$$(a + b)^2 = 4\,(\tfrac{1}{2})\,(b \cdot a) + c^2$$

$$a^2 + 2ab + b^2 = 2ab + c^2$$

$$a^2 + b^2 = c^2$$